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P116 Seminar 7 Experiment 1

Measuring the Birefringence of a Liquid Crystal

The birefringence of the liquid crystal pentyl-cyanobiphenyl (5CB) was measured as a function of temperature by placing the sample between perpendicular or parallel polarizers. The birefringence was about 0.17 at 27° C, dropping to about 0.12 at 35° C; a linear fit to our data shows a drop of about $0.8 \pm 0.1\%$ per degree Celcius.

Introduction

Due to their shape and structure, nematic liquid crystals behave as positive uniaxial birefringent crystals. The birefringence of a liquid crystal is dependent on applied voltage, wavelength, and temperature; in this experiment, the temperature dependence was studied by using the crystal as a retardation plate.

Theory

The Jones matrix for a retardation plate with its slow axis vertical is

$$e^{i\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}. \tag{1}$$

Applying rotation matrices to this Jones matrix, we arrive at the Jones matrix for a retardation plate with its slow axis oriented at an angle γ from the vertical:

$$\begin{aligned} A &= e^{i\pi/4}\begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}\begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} & (2) \\ &= e^{i\pi/4}\begin{pmatrix} \cos^2\gamma + e^{i\delta}\sin^{\gamma} & \frac{1}{2}\sin 2\gamma(1-e^{i\delta}) \\ \frac{1}{2}\sin 2\gamma(1-e^{i\delta}) & \sin^2\gamma + e^{i\delta}\cos^2\gamma \end{pmatrix}. & (3) \end{aligned}$$

Applying this matrix for the Jones vector for vertically polarized light, we have

$$A\begin{pmatrix} 0 \\ \sqrt{I_0} \end{pmatrix} = e^{i\pi/4}\sqrt{I_0}\begin{pmatrix} \frac{1}{2}\sin 2\gamma(1-e^{i\delta}) \\ \sin^2\gamma + e^{i\delta}\cos^2\gamma \end{pmatrix}, \tag{4}$$

where the square comes from the fact that the Jones vectors are proportional to electric fields rather than irradiances. Let us now define $I_\perp$ as the irradiance of vertically polarized light passing through the above retardation plate followed by a horizontal polarizer and $I_\parallel$ as the irradiance of the same light passing through a vertical polarizer. These two irradiances are given by the absolute squares of the components of the above Jones vector, and so we have

$$\begin{aligned} I_\perp &= I_0\left|\frac{1}{2}\sin 2\gamma(1-e^{i\delta})\right|^2 = \frac{I_0}{2}\sin^2 2\gamma(1-\cos\delta) \\ &= I_0\sin^2 2\gamma\sin^2\frac{\delta}{2} & (5) \end{aligned}$$

and

$$
\begin{aligned}
I_{\parallel} &= I_0 \left|\sin^2\gamma + e^{i\delta}\cos^2\gamma\right|^2 = I_0\left(\sin^4\gamma + \frac{1}{2}\sin^2 2\gamma\cos\delta + \cos^4\gamma\right) \\
&= I_0\left((\sin^2\gamma + \cos^2\gamma)^2 - \frac{1}{2}\sin^2 2\gamma + \frac{1}{2}\sin^2 2\gamma - \sin^2 2\gamma\sin^2\frac{\delta}{2}\right) \\
&= I_0\left(1 - \sin^2 2\gamma\sin^2\frac{\delta}{2}\right). \qquad (6)
\end{aligned}
$$

Note that the maximum of $I_{\parallel}$ occurs at I_0, the intensity of the initial vertically polarized light, and the mimimum occurs at $I_0(1 - \sin^2(\delta/2))$. Thus, the retardation δ can be determined by measuring these extrema. As Hecht explains in Section 8.7, the birefringence of a crystal, Δn, is related to the retardation by $\delta = k_0 d\Delta n$, where d is the thickness of the retardation plate. We can thus write Δn as

$$\Delta n = \frac{\delta\lambda_0}{2\pi d}. \qquad (7)$$

Procedure

The setup for the experiment is shown in Figure (1). The signal from the HeNe laser was first passed through a cheap polarizer to reduce the irradiance. It was then passed through a vertical polarizer and a chopper before reaching the liquid crystal sample. The sample serves as a retardation plate, and its angle was manually adjusted. The laser then passed through a horizontal or vertical polarizer before being measured by a detector.

A greater irradiance hitting the pn junction detector results in more carriers being promoted to the conduction band, resulting in a higher current. This current is amplified to a voltage which is measured by the lock-in amplifier. Because the frequency from the chopper was connected to the lock-in detector, inputs of other frequencies could be ignored, allowing us to perform the experiment with normal lighting. In this experiment, the voltage measured is proportional to the irradiance of the beam hitting the detector. We will therefore refer to irradiance in units of voltage, even though it is really energy/(area*time).

From Equation (6), we see that only the maximum and minimum signals of parallel (vertically) polarized light need be measured to determine the birefringence. For the first temperature, however, a number of measurements were taken of both $I_{\parallel}$ and $I_{\perp}$ to verify their functional forms. This was done at 34° C.

We then decreased the temperature in increments of one degree to 27° C, measuring only the average maximum and minimum signals of $I_{\parallel}$ at each temperature. The average maximum signal was obtained by averaging the maxima that occurred around 0° and ±90° and the average minimum signal was obtained by averaging the mimima that occured around ±45°.

Measurements were repeated at three temperatures after making adjustments to the orientation of the sample, since the light may have been striking different portions of the sample as it was rotated.

Results and Discussion

Figures (2) and (3) show the irradiances of parallel and perpendicularly polarized light as a function of the angle of the liquid crystal with respect to the vertial, measured at 34° C. From Eq. (6), the data for the parallel light was fit to the functional form $a(1 - b\sin(2x + c))$. Note that x is not exactly γ, since the sample was not exactly vertical. From this fit we see that I_0 is 275 ± 9 mV, and $\sin^2(\delta/2)$ is 0.93 ± 0.04, which gives a birefringence of $\Delta n = 0.117 \pm 0.005$. As a check, we note that using these numbers, $I_0 \sin^2(\delta/2) = 257 \pm 19$ mV, which is within the curve fit errors of the result for the perpendicular light, 273 ± 3 mV.

The rest of our data is seen in Table (1). By measuring the maximum and minimum for the parallel light, we could determine the birefringence at each temperature. Note that because the inverse sine function is multivalued, we had to determine which value to pick based on physical grounds; namely, that the birefringence is smallest at 35° C and increases as temperature decreases. To calculate the birefringence, we plugged the thickness of the liquid crystal sample (2.07μm, according to Peter), the wavelength of light (633 nm), and the retardation into Eq. (7).

Table 1: Measured Signals and Resulting Birefringences

T (°C)	ave. max. (mV)	ave. min. (mV)	$\delta/2$ (degrees)	corrected $\delta/2$ (rad)	Δn
35*	297 ± 4	80 ± 8	58.85 ± 0.07	1.027 ± 0.001	0.1000 ± 0.0001
33	279 ± 40	6 ± 2	82 ± 1	1.43 ± 0.02	0.139 ± 0.002
32	277 ± 30	1.5 ± 0.5	86 ± 2	1.50 ± 0.04	0.146 ± 0.004
31	276 ± 20	1.01 ± 0.07	86.5 ± 0.8	1.5 ± 0.2	0.147 ± 0.001
31*	296 ± 2	1.3 ± 0.3	86 ± 11	1.50 ± 0.04	0.15 ± 0.02
30	277 ± 6	2 ± 1	85 ± 2	1.66 ± 0.04	0.161 ± 0.004
29	276 ± 5	4 ± 3	83 ± 2	1.70 ± 0.04	0.165 ± 0.004
28	275 ± 10	7 ± 4	80 ± 1	1.74 ± 0.02	0.169 ± 0.002
27	278 ± 20	12 ± 6	80 ± 1	1.78 ± 0.02	0.173 ± 0.002
27*	286 ± 8	14 ± 2	77.0 ± 0.3	1.780 ± 0.004	0.1750 ± 0.0005

*Data taken after adjusting sample orientation.

The birefringence is plotted as a function of temperature in Figure (4). From the linear fit, it is seen that Δn drops approximately $0.8 \pm 0.1\%$ for each °C increase. This is twice as high as the claim by Hecht in Section 8.12 that the birefringence of a liquid crystal usual decreases about 0.4% per °C increase.

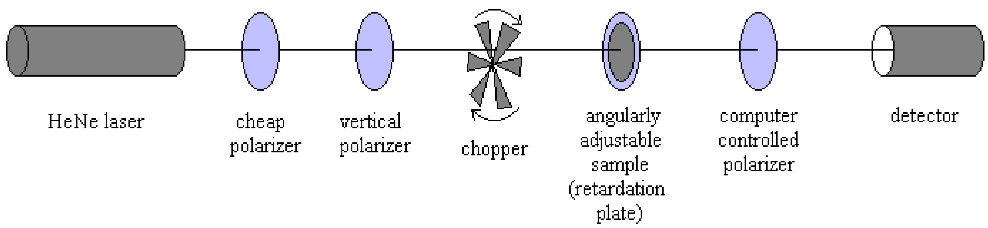

Figure 1: The signal from the HeNe laser was first passed through a cheap polarizer to reduce the irradiance. It was then passed through a vertical polarizer and a chopper before reaching the liquid crystal sample. The sample serves as a retardation plate, and its angle was manually adjusted. The laser then passed through a horizontal or vertical polarizer before being measured by a detector.

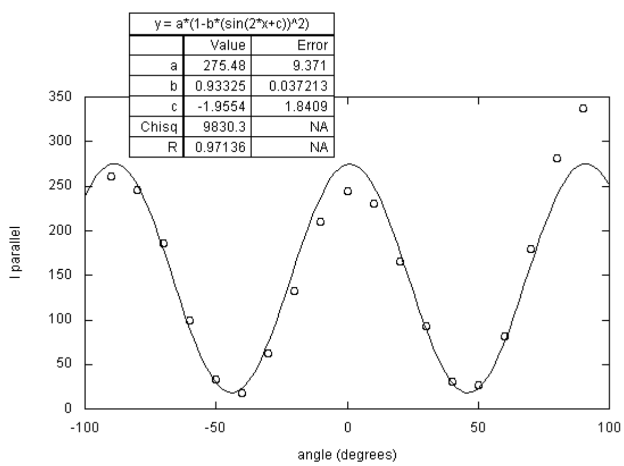

Figure 2: Irradiance of vertically polarized light as a function of the angle of the liquid crystal with respect to the vertical. This angle is not exactly γ, since the sample was not exacly vertical, and so an extra degree of freedom is allowed in the curve fit. These measurements were taken at 34° C.

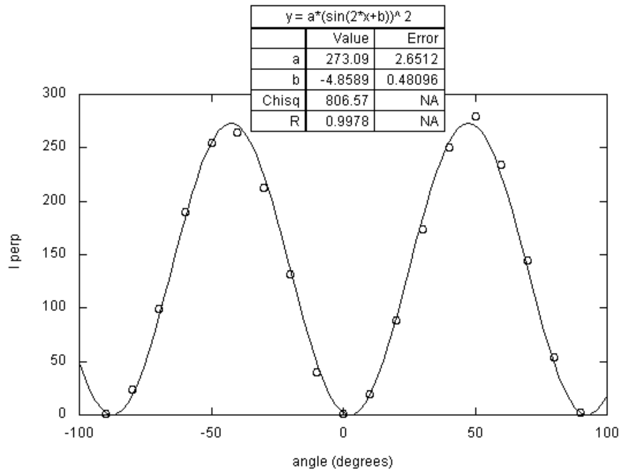

Figure 3: Irradiance of horizontally polarized light as a function of the angle of the liquid crystal with respect to the vertical. These measurements were taken at 34° C.

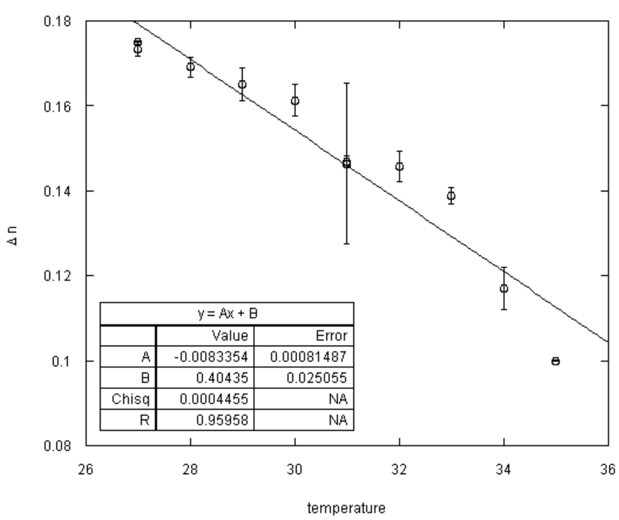

Figure 4: Birefringence as a function of temperature for a liquid crystal sample, with a linear fit. Δn drops approximately 0.8% for each °C increase.