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Physics 116 - Seminar 4

Dispersion in Prisms

Cauchy's equation (which is a special case of Sellmeier's equation, as I showed two weeks ago) gives $n(\lambda)$ when considering wavelengths away from resonance regions:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots . \tag{1}$$

These constants are generally determined by fitting empirical data. The **dispersion** of a material is defined as $dn/d\lambda$, which is approximately

$$\frac{dn}{d\lambda} = -\frac{2B}{\lambda^3} - \frac{4C}{\lambda^5}. \tag{2}$$

Note that dispersion is different from the **deviation** for a prism described in Hecht. Large dispersion means a large angle between where the light would have been if the prism were not in its path and where it ends up after passing through the prism. Large deviation means a large angle between light of different wavelengths after passing through the prism.

The **dispersive power** of a prism is the ratio of the dispersion to the deviation:

$$\Delta = \frac{\mathcal{D}}{\delta} = \frac{n_B - n_R}{n_Y - 1}, \tag{3}$$

where n_B is the index of refraction for 492-455 nm blue light, n_Y is for 597-577 nm yellow light, and n_R is for 780-622 nm red light.

As the wavelength difference between components of light incident on a prism decreases, the ability of the prism to resolve them begins to fail. Using the pictured geometry and Rayleigh's criterion, which gives the minimum resolvable separation between two wavefronts as $\Delta\alpha = \lambda/d$, we find a **resolving power** of

$$\mathcal{R} = \frac{\lambda}{(\Delta\lambda)_{min}} = b\frac{dn}{d\lambda}, \tag{4}$$

where $(\Delta\lambda)_{min}$ is the minimum wavelength separation permissible for resolvable images.