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Physics 114 - Seminar 13

The Renormalization Group Approach

"One cannot write a renormalization group cookbook."

—Kenneth G. Wilson, recipient of the 1982 Nobel Prize in Physics for RG theory

General RG Step	1D Ising Model Example
Find the partition function for your system.	$Z = \sum_{s_1,s_2,\dots,s_n} e^{(s_1 s_2 + s_2 s_3 + \cdots)J/kT}$
Write the partition function in terms of a coupling constant that tells you how strongly the particles interact.	$K \equiv J/kT$
Remove a finite fraction of the degrees of freedom by averaging (summing) over them.	$Z(K,N) = \sum_{s_1,s_2,\dots,s_n} e^{K(s_1 s_2 + s_2 s_3)} \times e^{K(s_3 s_4 + s_4 s_5)}$ $= \sum_{s_1,s_3,s_5,\dots,s_{n-1}} \left[e^{K(s_1+s_3)} + e^{-K(s_1+s_3)}\right] \times \left[e^{K(s_3+s_5)} + e^{-K(s_3+s_5)}\right] \times \cdots$
Cast your partially summed equation into a form that makes it look like the original system with a different coupling constant.	$f(K) e^{K' s_1 s_3} = e^{K(s_1+s_3)} + e^{-K(s_1+s_3)}$ $Z(K,N) = [f(K)]^{N/2} Z(K', N/2)$
Derive recursion relations. (Generally, this requires approximations, but not in the 1D Ising model.)	$K' = \frac{1}{2}\ln(\cosh 2K)$ $\zeta(K') = 2\zeta(K) - \ln\left[2(\cosh 2K)^{1/2}\right]$ (where $\ln Z(K,N) = N\zeta(K)$)
Iterate the recursion relations and look for notable behavior.	Easier to go from large length scale (small coupling constant) to small, so solve for: $K = \frac{1}{2}\ln\left[e^{2K'} + \left(e^{4K'} - 1\right)^{1/2}\right]$ $\zeta(K) = \frac{1}{2}\left[\zeta(K') + K' + \ln 2\right]$ Start with small K': $K' = 0.01 \Rightarrow \zeta(K') \approx \ln 2$ Iterate and plot results. (Baierlein Figure 16.8)
Wherever a divergence occurs, study the local behavior by a first-order Taylor expansion. Thereby extract critical exponents.	Not applicable: the only fixed points are at $K = 0$ and $K = \infty$, which means there is no possibility for a phase transition.