1 Motivation

Tropical Geometry combines three worlds: Geometry, Algebra, and Combinatorics. More specifically (and more technically), it's a combinatorial interpretation of Algebraic Geometry. While you may not understand what that means now, we'll develop what that means in this paper. First let's look at each of those three fields.

Geometry is the study of shapes and spaces. In it's earliest forms, it was the study of the relative positions of shapes and the study of measurements of length, area, and volume. One might use it to study spacetime, to learn about fractals, or to make computer graphics. **Algebra** has a deep connection to geometry, as the introduction of Cartesian coordinates gave mathematicians a way to characterize geometric objects using the language of symbols and equations used in algebra.

Algebra[1] is a much broader subject though; it's more than just solving for x or multiplying matrices together. Algebra is sometimes called the study of symmetry. In it we study the structure of mathematical objects, and how we can manipulate them. We are particularly interested in polynomials, an important aspect of mathematics that one runs into every day.

Consider the polynomial x^2+y^2-1. In studying polynomials, we are mostly interested in their roots. We want to find the values x and y that satisfy

$$x^2 + y^2 - 1 = 0.$$

Well this looks a lot like the equation for the unit circle, which is usually written as $x^2+y^2 = 1$. It turns out it is, but notice that we never spoke about circles in our question. We asked about the roots of a polynomial, the values where that polynomial is equal to zero—it turns out those values correspond to the points on the unit circle. With this, we introduce **Algebraic Geometry**.

We just that that the roots of a polynomial correspond to a geometric object. It turns out this is true in general: the roots of polynomials give rise to geometric structures, and the reverse is often true as well. Basically, we have the world of shapes and the world of polynomials, but we've found a bridge between the two of them. This means that we can translate problems in geometry into the language of algebra, and vice versa. For example, if I asked you to prove that two distinct ellipses meet in four points or fewer, you could just draw the picture and say that's that, but that's not a proof. It turns out it's difficult to prove in just the framework of geometry, but if we consider it's algebraic analogue, we can solve it quite easily (for someone who knows a little bit of abstract algebra).

With this in mind, we briefly describe **Combinatorics**, which is the study of discrete structures. We'll refine that definition later, but an important note is that algorithms can solve problems in combinatorics, and Tropical Geometry translates problems in algebraic geometry into the language of combinatorics...we'll expand more soon enough.

[1] Note that we actually refer to Abstract Algebra, but we use the terms interchangably.